The frictional drag force between quantum wells mediated by a fluctuating electromagnetic field

2001 J. Phys.: Condens. Matter 13 859

(http://iopscience.iop.org/0953-8984/13/5/307)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) 859–873 www.iop.org/Journals/cm PII: S0953-8984(01)17045-1

The frictional drag force between quantum wells mediated by a fluctuating electromagnetic field

A I Volokitin[1,2] **and B N J Persson**[1]

[1] Institut für Festkörperforschung, Forschungszentrum Jülich, D-52425, Germany
[2] Samara State Technical University, 443010 Samara, Russia

Received 11 September 2000

Abstract

We use the theory of the fluctuating electromagnetic field to calculate the frictional drag force between nearby two- and three-dimensional electron systems. The frictional drag results from coupling via a fluctuating electromagnetic field, and can be considered as the dissipative part of the van der Waals interaction. Unlike in other similar calculations for semiconductor two-dimensional systems, here retardation effects are included. We consider the dependence of the frictional drag force on the temperature T, electron density and separation d. We find that retardation effects become the dominating factor for high electron densities n_s, corresponding to thin metallic film, for separation $d > 10(\varepsilon/n_s)^{1/2}$, where ε is the dielectric constant, and suggest a new experiment to test the theory. The relation between friction and heat transfer is also briefly commented on.

1. Introduction

A great deal of attention has been recently devoted to double-layer systems in which two parallel quasi-two-dimensional (2D) subsystems (electron or hole gases) are separated by a potential barrier thick enough to prevent particles from tunnelling across it but allowing interaction between the particles on either side of it. Some time ago, Pogrebinskii and later Price [1] predicted that the Coulomb interaction between two 2D electron systems will induce a frictional drag force between the layers: a current in one film will induce a current in the adjacent film. The first frictional drag experiment was performed by Gramila *et al* for two electron layers [2,3] and by Sivan, Solomon and Shtrikman for an electron–hole system [4]. In these experiments a current is drawn in the first layer, while the second layer is an open circuit. Thus no dc current can flow in the second layer, but an induced electric field occurs that opposes the 'drag force' from the first layer. These experiments spurred on a large body of theoretical work both on electron–hole systems [5] and on electron–electron systems [6–15]. Most of this work focused on interlayer Coulomb interaction, the most obvious coupling mechanism and the one considered in the original theoretical papers [1], though the contributions due to an exchange of phonons between the layers have also been considered [3, 8, 9, 16]. The origin of Coulomb drag is quantum and thermal fluctuations of the charge and current densities and it

0953-8984/01/050859+15$30.00

can be considered as the dissipative part of the van der Waals interaction. The static aspects of the van der Waals interaction are well understood, and from the theory of Lifshitz [17] it is known that one must distinguish between two distance regimes:

(a) The non-retarded limit, where the separation between bodies d is small compared to the wavelengths $\lambda \sim c/\omega_0$, where ω_0 is a characteristic frequency of the charge fluctuation, and c the light velocity; the interaction is determined by the fluctuations in an instantaneous Coulomb field. For metal, $\omega_0 \sim \omega_p$, where ω_p is the plasma frequency.
(b) Retardation effects become important when $d > \lambda$. On the other hand, we have shown in references [18, 19] that, when calculating the dissipative part of the van der Waals interaction for two semi-infinite bodies in relative motion, retardation effects become important for $d > c/\omega_0$, where $\omega_0 \sim (\omega_p\tau)\omega_p$ and τ is the Drude relaxation time.

For $\omega_p \sim 10^{16}$ s^{-1} and $\tau \sim 10^{-14}$ s, retardation effects become important for very short distances $d > 1$ Å. However, for 2D systems there has been no investigation of the role of retardation effects in the frictional drag experiments. For large distances the retarded contribution to the frictional drag becomes important, and it is interesting to compare this contribution to the non-retarded contribution. To evaluate the retarded contribution from photon exchange we use the general theory of the fluctuating electromagnetic field developed by Rytov [20] and applied by Lifshitz [17] for studying the conservative part, and by us [18] for studying the dissipative part of the van der Waals interaction. In this approach the interaction between the bodies is mediated by the fluctuating electromagnetic field which is always present in the vicinity of any collection of atoms. Beyond the boundaries of a solid this field consists partly of travelling waves and partly of evanescent waves which are damped exponentially with the distance away from the surface of the body. The method that we use for calculating the frictional drag force between two nearby 2D systems is quite general, and is applicable to any body at arbitrary temperature. It takes into account retardation effects, which become important for large enough separation between the bodies.

We shall calculate the frictional stress $\sigma = \gamma v$ acting on the electrons in layer 1 due to the current density $J_2 = n_2 e v$ in layer 2, where n_2 is the carrier concentration (per unit area). If no current is allowed to flow in layer 1 (open circuit), an electric field E_1 develops whose influence cancels the frictional stress σ between the layers. The frictional stress $\sigma = \gamma v$ must equal the induced stress $n_1 e E_1$, so

$$\gamma = n_1 e E_1/v = n_1 n_2 e^2 E_1/J_2 = n_1 n_2 e^2 \rho_{12}$$

where the *transresistivity* $\rho_{12} = E_1/J_2$ is defined as the ratio of the induced electric field in the first layer to the driving current density in the second layer. The transresistivity is often interpreted in terms of a drag rate which, in analogy with a Drude model, is defined by $\tau_D^{-1} = \rho_{12} n_2 e^2/m^* = \sigma/n_1 m^* v$.

We find that for modulation-doped semiconductor quantum wells, retardation effects are not important under typical experimental conditions, supporting earlier calculations where retardation effects have always been neglected [5–15]. However, although some previous calculations for friction drag force between two-dimensional semiconductor systems are equivalent to ours, other approaches were very different. The present derivation offers an alternative insight and is more general. A striking new result that we find is that for systems with high 2D electron density, e.g. thin metallic films, retardation effects become crucial and in fact dominate the frictional shear stress σ. To test the theoretical predictions presented below, we therefore suggest performing experiments on thin metallic layers grown on insulating substrates and separated by thin insulating layers. For example, for two thin (∼monolayer) silver films separated by $d \sim 100$ Å, we estimate that the induced voltage U_1 in metal film

1, due to a current J_2 in layer 2, will be of order $U_1 \approx 10^{-8} U_2$, where U_2 is the driving voltage applied to metal film 2. Thus if $U_2 \approx 1$ V, the induced voltage will be of order 10 pV which it should be possible to detect experimentally. We note that the study of this problem is also of direct interest in the context of sliding friction, since the electronic friction probed when two metallic bodies slide relative to each other should be the same as the electronic drag force probed by the transresistivity measurement; see figure 1. The electronic sliding friction (usually called vacuum friction) has recently been invoked to explain experimental results for the damping of a small metal particle vibrating in the vicinity of a flat metal surface [21], but this explanation is controversial [19], and it is clear that independent studies of the electronic friction would be of great interest.

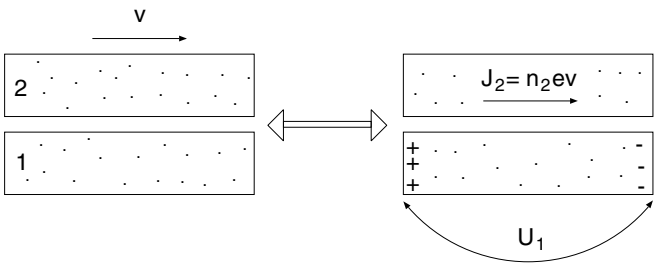

Figure 1. Left: a metallic block is sliding relative to the metallic substrate with the velocity v. An electronic frictional shear stress σ will act on the block (and on the substrate). Right: the shear stress σ can be measured if instead of sliding the upper block, a voltage U_2 is applied to the block resulting in a drift motion of the conduction electrons (velocity v). The resulting frictional stress σ on the substrate electrons will generate a voltage difference U_1 (proportional to σ) as indicated in the figure, which can be measured experimentally.

2. Calculation of the fluctuating electromagnetic field

We consider two parallel 2D electron layers separated by a distance d. We introduce two reference systems K and K', with coordinate axes xyz and $x'y'z'$. The xy- and $x'y'$-planes coincide with layer 1, with the x- and x'-axes pointing in the same direction, and the z- and z'-axes pointing toward layer 2. In the K-system both layers are at rest. Assume now that in layer 2 the conduction electrons move with the drift velocity v, corresponding to the current density $j_2 = n_2 ev$, while no current flows in layer 1. The reference system K' moves with velocity v along the x-axis relative to frame K. In the K'-frame there is no current density in layer 2, while the surrounding dielectric moves with velocity $-v\hat{\boldsymbol{x}}$. Following Lifshitz [17], to calculate the fluctuating field we shall use the general theory due to Rytov, which is described in his book [20]. This method is based on the introduction of a 'random' field in the Maxwell equations (just as, for example, one introduces a 'random' force in the theory of Brownian motion of a particle). In the K-system for $z < d$ for a monochromatic field (time factor $\exp(-\mathrm{i}\omega t)$) in a dielectric, non-magnetic medium, these equations are

$$\boldsymbol{\nabla} \times \boldsymbol{E} = \mathrm{i}\frac{\omega}{c}\boldsymbol{B} \tag{1}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} = -\mathrm{i}\frac{\omega}{c}\boldsymbol{D} + \frac{4\pi}{c}(\boldsymbol{j}_1 + \boldsymbol{j}_{1f})\delta(z) \tag{2}$$

where, following Rytov, we have divided the total current density $\boldsymbol{j}_{tot}$ in a layer into two parts, $\boldsymbol{j}_{tot} = \boldsymbol{j} + \boldsymbol{j}_f$, the fluctuating current density $\boldsymbol{j}_f$ associated with thermal and quantum

fluctuations and the current density $\boldsymbol{j}$ induced by the electric field $\boldsymbol{E}$:

$$j_\alpha(\boldsymbol{r}) = \int \mathrm{d}^2 r'\ \sigma_{\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}')E_\beta(\boldsymbol{r}') \tag{3}$$

where $\boldsymbol{r}$ is a 2D vector in the xy-plane, $\sigma_{1\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}')$ is the conductivity tensor in layer 1. $\boldsymbol{D}$, $\boldsymbol{H}$ and $\boldsymbol{B}$ are the electric displacement field, the magnetic and the magnetic induction fields, respectively. For a non-magnetic medium $\boldsymbol{B} = \boldsymbol{H}$ and $\boldsymbol{D} = \varepsilon\boldsymbol{E}$, where ε is the dielectric constant of the surrounded media. According to the fluctuation-dissipation theorem [22], the correlation function of the fluctuating current density $\boldsymbol{j}_f$, determining the average value of the product of the components of $\boldsymbol{j}_f$ at two different points in space, is given by the formulae

$$\begin{aligned}
&\langle j_{f\alpha}(\boldsymbol{r},\omega)j^*_{f\beta}(\boldsymbol{r}',\omega')\rangle = \langle j_{f\alpha}(\boldsymbol{r},\omega)j^*_{f\beta}(\boldsymbol{r}',\omega)\rangle_\omega\delta(\omega-\omega')\\
&\langle j_{f\alpha}(\boldsymbol{r},\omega)j^*_{f\beta}(\boldsymbol{r}',\omega)\rangle_\omega = \frac{\hbar\omega}{\pi}\left(\frac{1}{2}+n(\omega)\right)\mathrm{Re}\,\sigma_{\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}',\omega)
\end{aligned} \tag{4}$$

where the Bose–Einstein factor

$$n(\omega) = \frac{1}{\mathrm{e}^{\hbar\omega/k_BT}-1}$$

and where T is the temperature and $\mathrm{Re}\,\sigma_{\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}')$ is the real part of the conductivity. We represent the current density in the form of a Fourier integral:

$$\boldsymbol{j}(\boldsymbol{r}) = \int \mathrm{d}^2 q\ \boldsymbol{j}(\boldsymbol{q})\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \tag{5}$$

where $\boldsymbol{q}$ is a 2D vector in the xy-plane. For the Fourier components $\boldsymbol{j}_f(\boldsymbol{q})$, the correlation function corresponding to the spatial correlation (4) is

$$\langle j_{f\alpha}(\boldsymbol{q},\omega)j^*_{f\beta}(\boldsymbol{q}',\omega)\rangle_\omega = \frac{\hbar\omega}{4\pi^3}\left(\frac{1}{2}+n(\omega)\right)\mathrm{Re}\,\sigma_{\alpha\beta}(\boldsymbol{q},\omega)\delta(\boldsymbol{q}-\boldsymbol{q}') \tag{6}$$

where

$$\sigma_{\alpha\beta}(\boldsymbol{q},\omega) = \int \mathrm{d}^2 r\ \sigma_{\alpha\beta}(\boldsymbol{r},\omega)\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}.$$

For the layers with the assumed isotropy in the xy-plane, the conductivity tensor can be written in the form

$$\sigma_{\alpha\beta}(\boldsymbol{q},\omega) = \frac{q_\alpha q_\beta}{q^2}\sigma_l(\boldsymbol{q},\omega) + \left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right)\sigma_t(\boldsymbol{q},\omega) \tag{7}$$

where $\sigma_t(\boldsymbol{q},\omega)$ and $\sigma_l(\boldsymbol{q},\omega)$ are the transverse and longitudinal conductivity of the layer.

After decomposition of the components of the electromagnetic field into a Fourier integral, the general solution of the Maxwell equations for $z < d$ can be written in the form

$$\boldsymbol{E} = \begin{cases} \displaystyle\int_{-\infty}^{\infty}\{\boldsymbol{v}\mathrm{e}^{\mathrm{i}pz}+\boldsymbol{w}\mathrm{e}^{-\mathrm{i}pz}\}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\,\mathrm{d}^2q & 0<z<d\\ \displaystyle\int_{-\infty}^{\infty}\boldsymbol{u}_1\mathrm{e}^{-\mathrm{i}pz}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\,\mathrm{d}^2q & z<0 \end{cases} \tag{8}$$

$$\boldsymbol{B} = \begin{cases} \displaystyle\int_{-\infty}^{\infty}\{([\boldsymbol{q}\times\boldsymbol{v}]+p[\hat{\boldsymbol{z}}\times\boldsymbol{v}])\mathrm{e}^{\mathrm{i}pz} & \\ \qquad +([\boldsymbol{q}\times\boldsymbol{w}]-p[\hat{\boldsymbol{z}}\times\boldsymbol{w}])\mathrm{e}^{-\mathrm{i}pz}\}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\,\mathrm{d}^2q & 0<z<d\\ \displaystyle\int_{-\infty}^{\infty}([\boldsymbol{q}\times\boldsymbol{u}_1]-p[\hat{\boldsymbol{z}}\times\boldsymbol{u}_1])\mathrm{e}^{-\mathrm{i}pz}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\,\mathrm{d}^2q & z<0 \end{cases} \tag{9}$$

where $\boldsymbol{v}$, $\boldsymbol{w}$ and $\boldsymbol{u}_1$ satisfy the transversality conditions

$$\boldsymbol{v}\cdot\boldsymbol{q}+pv_z=0 \qquad \boldsymbol{w}\cdot\boldsymbol{q}-pw_z=0 \qquad \boldsymbol{u}_1\cdot\boldsymbol{q}+pu_{1z}=0 \tag{10}$$

where

$$p=\sqrt{\left(\frac{\omega}{c}\right)^2\varepsilon-q^2} \tag{11}$$

and $\hat{\boldsymbol{z}}$ is a unit vector along the z-axis. We now decompose the electromagnetic field into s- and p-polarized waves. The electric field $\boldsymbol{E}$ is in the plane determined by the vectors $\hat{\boldsymbol{q}}=\boldsymbol{q}/q$ and $\hat{\boldsymbol{z}}$ for the p-polarized waves and perpendicular to this plane along the vector $\boldsymbol{n}=\hat{\boldsymbol{z}}\times\hat{\boldsymbol{q}}$ for s-polarized waves. The boundary conditions at $z=0$ for s- and p-polarized waves are determined by

$$\begin{aligned} &E_n(z=+0)=E_n(z=-0)\\ &\left.\frac{\mathrm{d}E_n}{\mathrm{d}z}\right|_{z=+0}-\left.\frac{\mathrm{d}E_n}{\mathrm{d}z}\right|_{z=-0}=-\frac{4\pi\mathrm{i}\omega}{c^2}(\sigma_{1t}(\boldsymbol{q},\omega)E_n+j_{f1n}) \end{aligned} \tag{12}$$

$$\begin{aligned} &E_q(z=+0)=E_q(z=-0)\\ &\left.\frac{\mathrm{d}E_q}{\mathrm{d}z}\right|_{z=+0}-\left.\frac{\mathrm{d}E_q}{\mathrm{d}z}\right|_{z=-0}=-\frac{4\pi\mathrm{i}p^2}{\varepsilon\omega}(\sigma_{1l}(\boldsymbol{q},\omega)E_q+j_{f1q}) \end{aligned} \tag{13}$$

where $E_q=\hat{\boldsymbol{q}}\cdot\boldsymbol{E}$, $E_n=\boldsymbol{n}\cdot\boldsymbol{E}$ and so on. From (12) and (13) we can obtain the following equations:

$$v_q+R_{1\mathrm{p}}w_q=-\frac{4\pi pj_{f1q}}{\varepsilon\omega(\epsilon_{1\mathrm{p}}+1)} \tag{14}$$

$$v_n+R_{1\mathrm{s}}w_n=-\frac{4\pi\omega j_{f1n}}{pc^2(\epsilon_{1\mathrm{s}}+1)} \tag{15}$$

where $v_q=\hat{\boldsymbol{q}}\cdot\boldsymbol{v}$ and so on, and

$$R_{1\mathrm{s\,(1p)}}=\frac{\epsilon_{1\mathrm{s\,(1p)}}-1}{\epsilon_{1\mathrm{s\,(1p)}}+1} \qquad \epsilon_{1\mathrm{s}}=\frac{4\pi\omega\sigma_{1t}}{pc^2}+1 \qquad \epsilon_{1\mathrm{p}}=\frac{4\pi p\sigma_{1l}}{\omega\varepsilon}+1.$$

The Maxwell equations in the K'-system for $z>0$ have the same form as (1), (2) with $\boldsymbol{j}\to\boldsymbol{j}_2$ and $\boldsymbol{j}_f\to\boldsymbol{j}_{f2}$. However, to first order in v/c the relations between $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{B}$, $\boldsymbol{H}$ are [23]

$$\boldsymbol{D}=\varepsilon\boldsymbol{E}-(\varepsilon-1)\frac{v}{c}\hat{\boldsymbol{x}}\times\boldsymbol{B} \tag{16}$$

$$\boldsymbol{H}=\boldsymbol{B}-(\varepsilon-1)\frac{v}{c}\hat{\boldsymbol{x}}\times\boldsymbol{E}. \tag{17}$$

Under a Lorentz transformation, with accuracy to the term linear in v/c, we have $\omega'=\omega-q_xv$ and $\boldsymbol{q}'=\boldsymbol{q}-\hat{\boldsymbol{x}}\omega v/c^2$. Note also that p is invariant under the Lorentz transformation, i.e. $p=p'$. It can be shown that the last terms in (16), (17) give rise only to a coupling between s- and p-polarized waves. However, it can be shown [18] that this coupling gives a correction $\sim(v/c)^2$ to the frictional drag force between the layers, so this term can be omitted. In the $\hat{K}'$ reference frame for $z>0$ the Maxwell equations have the same form (1) and (2) with $\boldsymbol{E}$ and $\boldsymbol{B}$ replaced by $\boldsymbol{E}'$ and $\boldsymbol{B}'$, the index 1 changed to 2, $\delta(z)$ replaced by $\delta(z-d)$. The field $\boldsymbol{E}'$ and $\boldsymbol{B}'$ are given by the same formulae (8), (9) with the $\boldsymbol{r}$-coordinate, ω, $\boldsymbol{q}$ changed to $\boldsymbol{r}'$, ω', $\boldsymbol{q}'$, and $\boldsymbol{v}$, $\boldsymbol{w}$ replaced by $\boldsymbol{v}'$, $\boldsymbol{w}'$ for $0<z<d$, and $\boldsymbol{u}_1$ replaced by $\boldsymbol{u}_2$ and a change

in the sign of p for $z > d$ (the waves now propagate along the positive z-direction). From the boundary conditions for the s- and p-polarized waves we get the equations

$$w'_{q'} + R_{2\mathrm{p}}(\boldsymbol{q}', \omega')\mathrm{e}^{2\mathrm{i}pd}v'_{q'} = -\frac{4\pi p j_{f2q'}\mathrm{e}^{\mathrm{i}pd}}{\varepsilon\omega'(\epsilon_{2\mathrm{p}}+1)} \tag{18}$$

$$w'_{n'} + R_{2\mathrm{s}}(\boldsymbol{q}', \omega')\mathrm{e}^{2\mathrm{i}pd}v'_{n'} = -\frac{4\pi\omega' j_{f2n'}\mathrm{e}^{\mathrm{i}pd}}{pc^2(\epsilon_{2\mathrm{s}}+1)}. \tag{19}$$

The relations between the fields in the reference frames K and K' are determined by the Lorentz transformation. As was shown in reference [18], such a Lorentz transformation gives terms of the order v/c which couple the s- and p-polarized waves, but this results in a contribution to the frictional drag of the order of $(v/c)^2$. Thus we can take this transformation in zero order in v/c, so obtaining $v'_{q'}(\omega') = v_{\boldsymbol{q}}(\omega)$, $v'_{n'}(\omega') = (\omega'/\omega)v_{\boldsymbol{n}}(\omega)$ and similar equations for $\boldsymbol{w}$. After the transformation, the solution of the system of equations (14), (15), (18), (19)) takes the form

$$v_{\boldsymbol{q}} = \frac{4\pi p}{\Delta_{\mathrm{p}}\varepsilon}\left[\frac{j_{f2q'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd}R_{1\mathrm{p}}(\boldsymbol{q}, \omega)}{(\epsilon_{2\mathrm{p}}(\boldsymbol{q}', \omega')+1)\omega'} - \frac{j_{f1\boldsymbol{q}}(\boldsymbol{q}, \omega)}{(\epsilon_{1\mathrm{p}}(\boldsymbol{q}, \omega)+1)\omega}\right] \tag{20}$$

$$w_{\boldsymbol{q}} = \frac{4\pi p}{\Delta_{\mathrm{p}}\varepsilon}\left[\frac{j_{f1\boldsymbol{q}}(\boldsymbol{q}, \omega)\mathrm{e}^{2\mathrm{i}pd}R_{2\mathrm{p}}(\boldsymbol{q}', \omega')}{(\epsilon_{1\mathrm{p}}(\boldsymbol{q}, \omega)+1)\omega} - \frac{j_{f2q'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd}}{(\epsilon_{2\mathrm{p}}(\boldsymbol{q}', \omega')+1)\omega'}\right] \tag{21}$$

$$v_{\boldsymbol{n}} = \frac{4\pi\omega}{\Delta_{\mathrm{s}}pc^2}\left[\frac{j_{f2n'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd}R_{1\mathrm{s}}(\boldsymbol{q}, \omega)}{(\epsilon_{2\mathrm{s}}(\boldsymbol{q}', \omega')+1)} - \frac{j_{f1\boldsymbol{n}}(\boldsymbol{q}, \omega)}{(\epsilon_{1\mathrm{s}}(\boldsymbol{q}, \omega)+1)}\right] \tag{22}$$

$$w_{\boldsymbol{n}} = \frac{4\pi\omega}{\Delta_{\mathrm{s}}pc^2}\left[\frac{j_{f1\boldsymbol{n}}(\boldsymbol{q}, \omega)\mathrm{e}^{2\mathrm{i}pd}R_{2\mathrm{s}}(\boldsymbol{q}', \omega')}{(\epsilon_{1\mathrm{s}}(\boldsymbol{q}', \omega)+1)} - \frac{j_{f2n'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd}}{(\epsilon_{2\mathrm{s}}(\boldsymbol{q}', \omega')+1)}\right] \tag{23}$$

$$v_z = -\frac{qv_q}{p} \qquad w_z = \frac{qw_q}{p} \tag{24}$$

where we have introduced the notation

$$\Delta_{\mathrm{p}} = 1 - \mathrm{e}^{2\mathrm{i}pd}R_{2\mathrm{p}}(\boldsymbol{q}', \omega')R_{1\mathrm{p}}(\boldsymbol{q}, \omega)$$
$$\Delta_{\mathrm{s}} = 1 - \mathrm{e}^{2\mathrm{i}pd}R_{2\mathrm{s}}(\boldsymbol{q}', \omega')R_{1\mathrm{s}}(\boldsymbol{q}, \omega).$$

3. Calculation of the frictional drag force between 2D systems

The frictional drag stress σ which acts on the conduction electrons in layer 1 can be obtained from the xz-component of the Maxwell stress tensor σ_{ij}, evaluated at $z = \pm 0$:

$$\sigma = \frac{1}{8\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega\,\{[\varepsilon\langle E_z E_x^*\rangle + \langle B_z B_x^*\rangle + \mathrm{c.c.}]_{z=+0} - [\cdots]_{z=-0}\}. \tag{25}$$

Here $\langle\cdots\rangle$ denotes statistical averaging over the fluctuating current densities. The averaging is carried out with the aid of (6). Note that the components of the fluctuating current density $\boldsymbol{j}_{f1}$ and $\boldsymbol{j}_{f2}$ refer to different layers, and are statistically independent, so the average of their product is zero. Expanding the electric field and magnetic induction in Fourier series we obtain

$$\sigma = \frac{1}{8\pi}\int \mathrm{d}\omega\,\mathrm{d}^2q\,\{[\varepsilon\langle E_z(\boldsymbol{q}, \omega)E_x^*(\boldsymbol{q}, \omega)\rangle + \langle B_z(\boldsymbol{q}, \omega)B_x^*(\boldsymbol{q}, \omega)\rangle + \mathrm{c.c.}]_{z=+0} - [\cdots]_{z=-0}\}. \tag{26}$$

For a given value of $\boldsymbol{q}$ it is convenient to express the components E_x and B_x in terms of the components along the vectors $\hat{\boldsymbol{q}}$ and $\boldsymbol{n}$:

$$E_x = (q_x/q)E_{\boldsymbol{q}} - (q_y/q)E_{\boldsymbol{n}} \tag{27}$$

$$B_x = (q_x/q)B_{\boldsymbol{q}} - (q_y/q)B_{\boldsymbol{n}}. \tag{28}$$

After substitution of expressions (27), (28) into (26) and taking it into account that the term which is proportional to q_y is equal to zero [18], we obtain

$$\sigma = \frac{1}{8\pi}\int \mathrm{d}\omega\, \mathrm{d}^2q\, \frac{q_x}{q}\{[\varepsilon\langle E_z(\boldsymbol{q},\omega)E^*_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle + \langle B_z(\boldsymbol{q},\omega)B^*_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle + \text{c.c.}]_{z=+0} - [\cdots]_{z=-0}\} \tag{29}$$

where

$$E_z(z=+0) = (v_z + w_z) = (q/p)(w_{\boldsymbol{q}} - v_{\boldsymbol{q}}) = (qp^*/|p|^2)(w_{\boldsymbol{q}} - v_{\boldsymbol{q}}) \tag{30}$$

$$E_z(z=-0) = u_{1z} = (q/p)u_{\boldsymbol{q}} = (q/p)(w_{\boldsymbol{q}} + v_{\boldsymbol{q}}) \tag{31}$$

$$E_{\boldsymbol{q}}(z=+0) = E_{\boldsymbol{q}}(z=-0) = v_{\boldsymbol{q}} + w_{\boldsymbol{q}} \tag{32}$$

$$B_z(z=+0) = (qc/\omega)(v_{\boldsymbol{n}} + w_{\boldsymbol{n}}) = B_z(z=-0) = (qc/\omega)u_{1\boldsymbol{n}} \tag{33}$$

$$B_{\boldsymbol{q}}(z=+0) = (pc/\omega)(w_{\boldsymbol{n}} - v_{\boldsymbol{n}}) \tag{34}$$

$$B_{\boldsymbol{q}}(z=-0) = (pc/\omega)u_{1\boldsymbol{n}}. \tag{35}$$

After substituting these expressions into formula (29) we obtain

$$\begin{aligned}\sigma = \frac{1}{4\pi}\int_0^{+\infty} \mathrm{d}\omega \int \mathrm{d}^2q\, q_x \Bigg(&\frac{\varepsilon}{|p|^2}[(p+p^*)(\langle|w_{\boldsymbol{q}}|^2\rangle - \langle|v_{\boldsymbol{q}}|^2\rangle \\ &- \langle|v_{\boldsymbol{q}} + w_{\boldsymbol{q}}|^2\rangle) + (p-p^*)\langle(v_{\boldsymbol{q}}w^*_{\boldsymbol{q}} - v_{\boldsymbol{q}}w^*_{\boldsymbol{q}})\rangle] \\ &+ \left(\frac{c}{\omega}\right)^2[(p+p^*)(\langle|w_{\boldsymbol{n}}|^2\rangle - \langle|v_{\boldsymbol{n}}|^2\rangle - \langle|v_{\boldsymbol{n}} + w_{\boldsymbol{n}}|^2\rangle) \\ &- (p-p^*)\langle(v_{\boldsymbol{n}}w^*_{\boldsymbol{n}} - v_{\boldsymbol{n}}w^*_{\boldsymbol{n}})\rangle]\Bigg)\end{aligned} \tag{36}$$

where we integrate only over positive values of ω, which gives an extra factor of two.

Substituting (20) and (24) into (36) and taking it into account that $p = p^*$ for $q < \omega/c$ and $p = -p^*$ for $q > \omega/c$, we obtain

$$\begin{aligned}\sigma = \frac{\hbar}{8\pi^3}\int_0^{\infty} \mathrm{d}\omega \int_{q<(\omega/c)\sqrt{\varepsilon}} \mathrm{d}^2q\, q_x \Bigg[&\frac{T_{1\mathrm{p}}(\omega)T_{2\mathrm{p}}(\omega - q_xv)(n(\omega - q_xv) - n(\omega))}{|1 - \mathrm{e}^{2\mathrm{i}pd}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega - q_xv)|^2} \\ &- \frac{T_{1\mathrm{p}}(\omega)(|1 - R_{2\mathrm{p}}(\omega - q_xv)|^2 + |1 - \mathrm{e}^{2\mathrm{i}pd}R_{2\mathrm{p}}(\omega - q_xv)|^2)(n(\omega) + 1/2)}{|1 - \mathrm{e}^{2\mathrm{i}pd}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega - q_xv)|^2}\Bigg] \\ &+ \frac{\hbar}{2\pi^3}\int_0^{\infty} \mathrm{d}\omega \int_{q>(\omega/c)\sqrt{\varepsilon}} \mathrm{d}^2q\, q_x\mathrm{e}^{-2|p|d} \\ &\times \frac{\mathrm{Im}\, R_{1\mathrm{p}}(\omega)\,\mathrm{Im}\, R_{2\mathrm{p}}(\omega - q_xv)}{|1 - \mathrm{e}^{-2|p|d}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega - q_xv)|^2}(n(\omega - q_xv) - n(\omega)) + [\mathrm{p} \to \mathrm{s}]\end{aligned} \tag{37}$$

where

$$T_{i\mathrm{p}}(\omega) = 1 - |R_{i\mathrm{p}}|^2 - |1 - R_{i\mathrm{p}}|^2 = \frac{16\pi\,\mathrm{Re}\,\sigma_{il}(\omega)p}{\omega\varepsilon|\epsilon_{il} + 1|^2}$$

$$T_{i\mathrm{s}}(\omega) = 1 - |R_{i\mathrm{s}}|^2 - |1 - R_{i\mathrm{s}}|^2 = \frac{16\pi\,\mathrm{Re}\,\sigma_{it}(\omega)\omega}{pc^2|\epsilon_{it} + 1|^2}.$$

The first integral in (37) is the contribution to the frictional drag force from propagating electromagnetic waves. The second term in (37) is derived from the evanescent field.

4. Some limiting cases

Consider distances $d \ll d_W \sim c\hbar/k_BT$ (at $T = 3$ K we have $d_W \sim 10^6$ Å). In this case we can neglect the first integral in (37), put $p \approx iq$ and extend the integral over $\boldsymbol{q}$ to the whole q-plane. Using these approximations, and in the linear approximation in the velocity v, the second integral in (37) can be written as [18]

$$\sigma = \frac{\hbar v}{2\pi^2}\int_0^\infty \mathrm{d}q\; q^3 \mathrm{e}^{-2qd}\int_0^\infty \mathrm{d}\omega\left(-\frac{\mathrm{d}n}{\mathrm{d}\omega}\right)\left\{\frac{\mathrm{Im}\,R_{1\mathrm{p}}(\omega)\,\mathrm{Im}\,R_{2\mathrm{p}}(\omega)}{|1-\mathrm{e}^{-2qd}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega)|^2}+[\mathrm{p}\to\mathrm{s}]\right\}. \tag{38}$$

Let us describe the 2D layers in the RPA approximation. For $q < k_F$ (corresponding to separations $d > k_F^{-1}$, where k_F is the Fermi wavevector of the degenerate electron gas system; for a 2D electron layer with electron density $n_s \approx 1.5\times 10^{11}$ cm^{-2}, $k_F = (2\pi n_s)^{1/2} \sim 10^6$ cm^{-1}) the transverse and longitudinal parts of the conductivity for a 2D electron layer can be written in the form [24, 25]

$$\sigma_l = \frac{\mathrm{i}\omega e^2 n_s}{q^2\epsilon_F}\left\{\frac{\omega\overline{u}}{(\omega+\mathrm{i}\gamma)\sqrt{\overline{u}^2-1}-\mathrm{i}\gamma\overline{u}}-1\right\} \tag{39}$$

$$\sigma_t = -\frac{2\mathrm{i}e^2 n_s\overline{u}(\sqrt{\overline{u}^2-1}-\overline{u})}{m^*(\omega+\mathrm{i}\gamma)} \tag{40}$$

where $\overline{u} = (\omega+\mathrm{i}\gamma)/qv_F$, $\gamma = 1/\tau$, $v_F = \hbar k_F/m^*$ is the Fermi velocity, τ is a relaxation time, $\epsilon_F = \hbar^2k_F^2/2m^*$ is the Fermi energy. In experiment [2, 3], $m^* = 0.067\; m_e$, $v_F = 1.6\times 10^7$ cm s^{-1}, $\epsilon_F \sim 60$ K and the mobility $\mu \sim 2\times 10^6$ cm^2 V^{-1} s^{-1}, so $\tau \sim 7.6\times 10^{-11}$ s. Let us divide the integration over $0 < q < \infty$ into the two parts $0 < q < \omega/v_F$ and $\omega/v_F < q < \infty$. In the first part of the integration, $\overline{u} > 1$ and, taking the limit $\overline{u} \gg 1$, we obtain in this limit the Drude formula for the conductivity

$$\sigma_l = \sigma_t = \frac{\mathrm{i}e^2 n_s}{m^*(\omega+\mathrm{i}\gamma)}. \tag{41}$$

In the second part of the integration, $\overline{u} < 1$ and, taking the limit $\overline{u} \ll 1$, we obtain

$$\sigma_l = \frac{\omega e^2 n_s}{q^2\epsilon_F}(u-\mathrm{i}) \tag{42}$$

$$\sigma_t = \frac{e^2 n_s v_F}{\epsilon_F q}$$

where we put γ equal to zero because it makes only a small contribution in this limit.

Let us consider the case of small separation d when $a = (2k_BTd/\hbar v_F) < 1$. Introducing the dimensionless variables $q = x/2d$ and $\omega = (k_BT/\hbar)y$, we obtain in this limit for $ay < x < \infty$

$$R_{\mathrm{p}} = \frac{\lambda_{\mathrm{p}}(x+\mathrm{i}ay)}{x^2+\lambda_{\mathrm{p}}(x+\mathrm{i}ay)} \approx 1-\frac{x^2}{\lambda_{\mathrm{p}}(x+\mathrm{i}ay)} \tag{43}$$

$$R_{\mathrm{s}} = \frac{\mathrm{i}\lambda_{\mathrm{s}}y}{\mathrm{i}\lambda_{\mathrm{s}}y-x^2} \tag{44}$$

and for $0 < x < ay$

$$R_{\rm p} = \frac{\lambda'_{\rm p} x}{\lambda'_{\rm p} x - 2y^2 - 2{\rm i}y\delta} \tag{45}$$

$$R_{\rm s} = \frac{\lambda'_{\rm s} y}{2xy + \lambda'_{\rm s} y + 2{\rm i}x\delta} \tag{46}$$

where

$$\lambda_{\rm p} = \frac{8\pi e^2 n_s d}{\varepsilon m^* v_F^2} \qquad \lambda_{\rm s} = 8\pi a d\left(\frac{e^2 n_s}{m^* c^2}\right)$$

$$\lambda'_{\rm p} = \frac{2\pi n_s e^2}{\varepsilon m^* d}\left(\frac{\hbar}{k_B T}\right)^2 \qquad \lambda'_{\rm s} = \frac{8\pi n_s e^2 d}{m^* c^2}.$$

We note that expression (45) has a pole at

$$\omega^2 = \frac{2\pi n_s e^2}{\varepsilon m^*} q \tag{47}$$

which corresponds to the plasmon excitations [27]. After substituting (43)–(46) in (38) we obtain for the frictional drag rate

$$\tau_{D{\rm p}}^{-1} \approx 0.2360 \frac{(kT)^2}{\hbar \epsilon_F (q_{TF} d)^2 (k_F d)^2} + 10\left(\frac{k_B T}{\epsilon_F}\right)^5 \left(\frac{k_B T}{\epsilon_{TF}}\right)^2 \gamma \tag{48}$$

$$\tau_{D{\rm s}}^{-1} \approx 3.3 \times 10^{-5}\left(\frac{k_B T}{m^* c^2}\right)\left(4\frac{k_B T}{\hbar} + \gamma\right) \tag{49}$$

where $\tau_{D{\rm p}}^{-1}$ and $\tau_{D{\rm s}}^{-1}$ are the contributions from s- and p-polarized waves, respectively, $q_{TF} = 2e^2 m^*/\hbar^2 \varepsilon$ is the single-layer Thomas–Fermi screening wavevector, $\epsilon_{TF} = \hbar^2 q_{TF}^2/2m^*$. The first term in (48) agrees with the results of Gramila *et al* [3] and Persson and Zhang [26]. From comparison of (48) and (49) it follows that for

$$n_s < n_c \sim 10^2 \left(\frac{m^* k_B T}{\pi \hbar^2}\right)\left(\frac{\varepsilon^4 \hbar^2 k_B T}{m^* e^4}\right)^{1/5} \tag{50}$$

the contribution from p-polarized waves exceeds the contribution from s-polarized waves for all distances $d < \hbar v_F/k_B T$. However, for $n_s > n_c$ the contribution from s-polarized waves will dominate for $d > 10(\varepsilon/n_s)^{1/2}$. For example, for $T = 3$ K and for the conditions of the experiment of references [2, 3], $n_c \sim 10^{12}$ cm^{-2}, and we find that in this case the retardation effects are small. However, retardation effects are important for high electron densities. For example, assuming that $\varepsilon = 10$ and $n_s \approx 10^{15}$ cm^{-2}, which corresponds to about one monolayer of silver, we find that the contribution to frictional drag from the retardation effects will dominate for $d > 10^2$ Å. This is illustrated in figure 2(a) which shows the shear stress when the relative velocity $v = 1$ m s^{-1}. We have performed calculations with $\varepsilon = 1$ (the result for an arbitrary ε can be obtained from these calculations by using the scaling $\tau_{D{\rm p}}^{-1} \sim \varepsilon^2$ and $\tau_{D{\rm s}}^{-1}$ is independent of ε) and $n_s \approx 10^{15}$ cm^{-2} for two different temperatures, $T = 273$ K and 77 K, and the s- and p-wave contributions are shown separately. From these calculations we obtain that the s-wave contribution dominates for $d > 15$ Å with $\varepsilon = 1$ and for $d > 45$ Å with $\varepsilon = 10$. In figure 2(b) we show the same quantity for two quantum wells at $T = 3$ K and with $n_s = 1.5 \times 10^{11}$ cm^{-2}, $m^* = 0.067\, m_e$, $v_F = 1.6 \times 10^7$ cm s^{-1} and $\tau = 7.6 \times 10^{-11}$ s, and with $\varepsilon = 1$. In this case the p-wave contribution dominates for $d < 1000$ Å with $\varepsilon = 1$ and for $d < 3 \times 10^3$ Å with $\varepsilon = 10$.

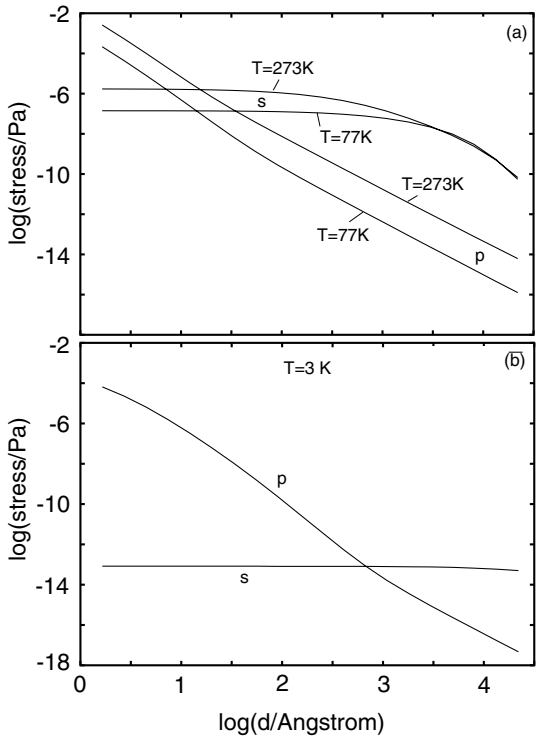

Figure 2. The shear stress as a function of the distance d between the surfaces (the log function is to the base 10). (a) For one-monolayer silver films for two different temperatures. The s- and p-wave contributions are shown separately. In the calculation, $\tau = 4 \times 10^{-14}$ s and 20×10^{-14} s for $T = 273$ K and 77 K, respectively. We have assumed $n_s = 1.05 \times 10^{19}$ m^{-2}, $m^* = m_e$ and $v_F = 1.4 \times 10^6$ m s^{-1}. (b) For quantum wells at $T = 3$ K. In the calculation, $\tau = 7.6 \times 10^{-11}$ s, $n_s = 1.5 \times 10^{15}$ m^{-2}, $m^* = 0.067\, m_e$ and $v_F = 1.6 \times 10^5$ m s^{-1}.

Let us estimate the voltage U_1 induced in a thin silver film (layer 1; open circuit) when a current flows in another parallel silver film (layer 2). A voltage difference of order 1 pV can be measured with standard equipment, so if U_1 is of the order 1 pV or larger, it may be possible to probe retardation effects with this experimental set-up. If L denotes the length of the metallic films (assumed identical) in the direction of the driving current, then $U_1 = LE_1$ and $U_2 = LE_2 = LJ_2/\sigma_2$ where $\sigma_2 = n_2e^2\tau_2/m^*$ is the conductivity (τ_2 is a Drude relaxation time and m^* the electron effective mass). Thus, using the equation (see the introduction) $\gamma = n_1n_2e^2E_1/J_2$ with $E_1/J_2 = U_1/(\sigma_2U_2) = (U_1/U_2)m^*/(n_2e^2\tau_2)$ gives $U_1 = (\gamma\tau/m^*n_1)U_2$. In a typical case, $\tau = 4 \times 10^{-14}$ s and $n_1 \approx 10^{15}$ cm^{-2}, and from figure 2(a), $\gamma \approx 10^{-6}$ N s m^{-2}, giving $U_1 \approx 10^{-8}\, U_2$. Thus if the applied voltage $U_2 \approx 1$ V, the induced voltage would be of order 10 pV, which it should be possible to measure.

5. The frictional drag force between 3D systems

For high electron densities, when the thickness of the layers $h \gg n^{-1/3}$, where n is a volume electron density, the electrons behave as in 3D systems. It was shown in reference [18] that for 3D systems the frictional drag stress is also given by formula (38), where the electromagnetic

reflection coefficients are

$$R_{i\mathrm{p}} = \frac{\varepsilon_i p - \varepsilon s_i}{\varepsilon_i p + \varepsilon s_i} \qquad R_{i\mathrm{s}} = \frac{p - s_i}{p + s_i} \tag{51}$$

where ε_i is the complex dielectric constant for layer i, and

$$s_i = \sqrt{\frac{\omega^2}{c^2}\varepsilon_i - q^2}. \tag{52}$$

Consider two identical 3D layers described by the dielectric function

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}/\tau)} \tag{53}$$

where τ is the Drude relaxation time and ω_p the plasma frequency. For $k_B T > \hbar\gamma$, for small frequencies and $d < (c/\omega_p)(\hbar\gamma/k_B T)^{1/2}$,

$$\mathrm{Im}\, R_{\mathrm{p}} \approx \frac{2\varepsilon\omega}{\omega_p^2 \tau} \qquad \mathrm{Re}\, R_{\mathrm{p}} \approx 1$$

$$\mathrm{Re}\, R_{\mathrm{s}} \approx 0 \qquad \mathrm{Im}\, R_{\mathrm{s}} \approx 4\left(\frac{\omega_p}{c}\right)^2 \frac{\omega}{\gamma q^2} \qquad \text{for } q^2 > \left(\frac{\omega_p}{c}\right)^2 \frac{\omega}{\gamma}.$$

Then, taking into account that for 3D systems $n_s = nh$, equation (38) gives

$$\tau_{D\mathrm{p}}^{-1} = 13.32 \frac{(\varepsilon k_B T)^2}{\hbar \epsilon_F (k_F d)^2 (k_{TF} d)^2 (k_F h)(\omega_p \tau)^2} \tag{54}$$

$$\tau_{D\mathrm{s}}^{-1} = \frac{e^2 (k_B T)^2 (\omega_p \tau)^2}{8\pi\hbar h (m^* c^2)^2} \tag{55}$$

where $k_{TF}^2 = 6\pi n e^2/\epsilon_F$ is the 3D Thomas–Fermi screening wavevector and $k_F = (3\pi^2 n)^{1/3}$ is the 3D Fermi wavevector. From comparison of (54) with (55) we conclude that for $d > \varepsilon^{1/2}(c/\omega_p)(\omega_p\tau)^{-1}$, the s-wave contribution exceeds the p-wave contribution. For 'normal' metal at room temperature, $\omega_p \sim 10^{16}$ s^{-1}, $\omega_p\tau \sim 100$, so the s-wave contribution dominates for $d > 10$ Å with $\varepsilon = 10$. Thus, in the case of the dissipative van der Waals interaction between 3D bodies, retardation effects become important for much shorter distances than for the conservative one, where the retardation effects become important for $d > c/\omega_p$ [17].

Figure 3 shows the calculated frictional shear stress for two semi-infinite silver bodies moving with the relative velocity $v = 1$ m s^{-1} parallel to the flat surfaces. Results are shown for the s- and p-wave contributions, where in the latter case we have taken into account non-local effects (the dashed lines show the result when the local (long-wavelength) dielectric function is used). Results are shown for two different temperatures, $T = 70$ K and 300 K, and the observed temperature dependences reflect that of the temperature prefactor T^2 in the expression for the shear stress as well as the temperature dependence of the Drude relaxation time τ.

6. Relation between friction and heat transfer

The frictional shear stress studied above is closely related to the heat transfer from one solid to another when the solids have different temperatures. For large separation, the heat transfer is given by Stefan's law:

$$J_z = \frac{\pi^2 k_B^4}{60\hbar^3 c_0^2}(T_1^4 - T_2^4) \tag{56}$$

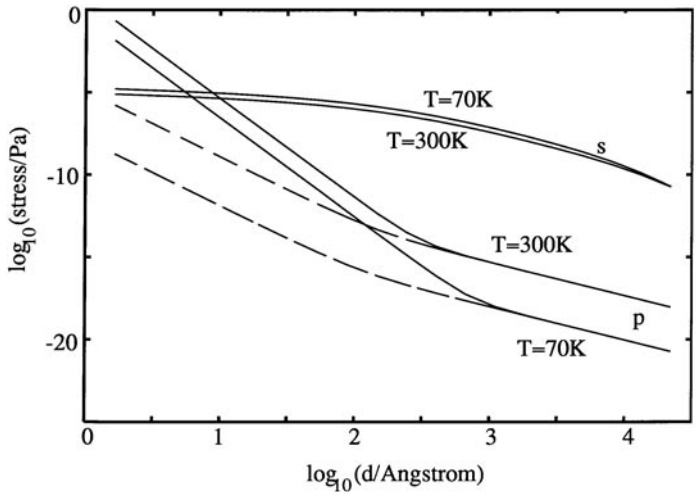

Figure 3. The shear stress as a function of the distance d between the surfaces of two semi-infinite silver bodies. The s- and p-wave contributions are shown separately and for two different temperatures, $T = 70$ K and 300 K. The p-wave contribution has been calculated using a local dielectric function (dashed lines) and using a theory which takes into account non-locality within the jellium model (solid line).

where T_1 and T_2 are the temperatures of solids 1 and 2, respectively. This formula corresponds to emission of real photons. However, for small separation d it is possible for the evanescent near field to transfer energy from one solid to the other. This corresponds to photon tunnelling. In general, the heat flux (energy flow per unit area and unit time) is given by a formula very similar to that for the frictional stress [28–30]:

$$J_z = \frac{\hbar}{8\pi^3}\int_0^\infty \mathrm{d}\omega\,\omega \int_{q<\omega/c} \mathrm{d}^2q \left[\frac{(1-|R_{1\mathrm{p}}(\omega)|^2)(1-|R_{2\mathrm{p}}(\omega)|^2)(n_1(\omega)-n_2(\omega)}{|1-\mathrm{e}^{2\mathrm{i}pd}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega)|^2}\right] + \frac{\hbar}{2\pi^3}\int_0^\infty \mathrm{d}\omega\,\omega \int_{q>\omega/c} \mathrm{d}^2q\,\mathrm{e}^{-2|p|d} \times \frac{\mathrm{Im}\,R_{1\mathrm{p}}(\omega)\,\mathrm{Im}\,R_{2\mathrm{p}}(\omega)}{|1-\mathrm{e}^{-2|p|d}R_{1\mathrm{p}}(\omega)R_{2\mathrm{p}}(\omega)|^2}(n_1(\omega)-n_2(\omega)) + [\mathrm{p}\to\mathrm{s}] \tag{57}$$

where

$$n_1(\omega) = (\mathrm{e}^{\hbar\omega/k_BT_1}-1)^{-1} \tag{58}$$

is the Bose–Einstein factor of solid 1 and similarly for n_2. Figure 4(a) shows the heat transfer between two semi-infinite silver bodies separated by the distance d and at the temperatures $T_1 = 273$ K and $T_2 = 0$ K. The s- and p-wave contributions are shown separately, and the p-wave contribution has been calculated using non-local optics (the lower solid line shows the result obtained using local optics). It is remarkable how important the s contribution is even for short distances. The detailed distance dependence of J_z has been studied by Polder and Van Hove [28] within the local optics approximation, and the analysis will not be repeated here. The non-local optics contribution to $(J_z)_\mathrm{p}$, which is important only for $d < l$ (where l is the electron mean free path in the bulk), is easy to calculate for free-electron-like metals. The non-local surface contribution to Im R_p is given by [26]

$$(\mathrm{Im}\,g)_\mathrm{surf} = 2\xi\frac{\omega}{\omega_p}\frac{q}{k_F}$$

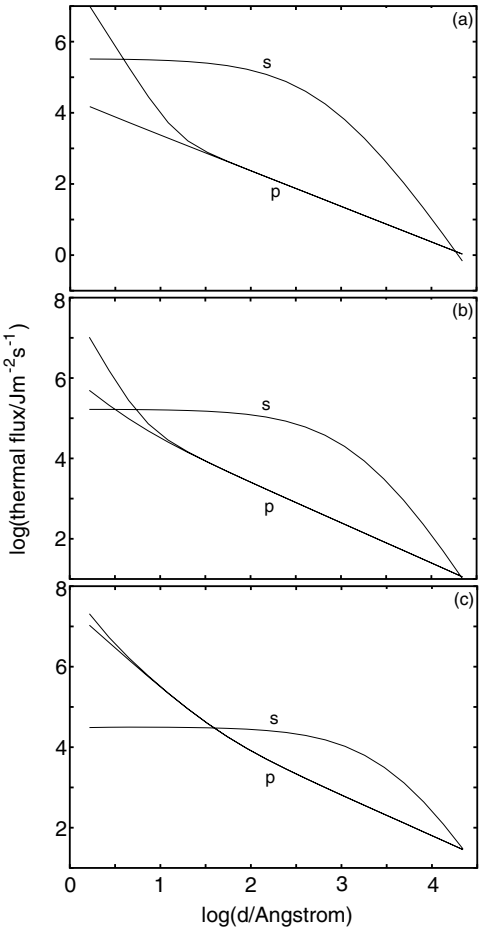

Figure 4. (a) The heat-transfer flux between two semi-infinite silver bodies, one at temperature $T_1 = 273$ K and another at $T_2 = 0$ K. (b) The same as (a) except that we have reduced the Drude electron relaxation time τ for solid 1 from a value corresponding to a mean free path $v_F\tau = l = 560$ Å to 20 Å. (c) The same as (a) except that we have reduced l to 3.4 Å.

where $\xi(q)$ depends on the electron density parameter r_s but typically $\xi(0) \sim 1$. Using this expression for Im R_p in (57) gives the (surface) contribution:

$$J_\mathrm{surf} \approx \frac{\xi^2 k_B^4}{\omega_p^2 k_F^2 d^4 \hbar^3}(T_1^4 - T_2^4).$$

Note from figure 4(a) that the local optics contribution to $(J_z)_\mathrm{p}$ depends nearly linearly on $1/d$ in the distance interval studied, and that this contribution is much smaller than the s-wave contribution. Both of these observations differ from those of reference [29], where it is stated that the s contribution can be neglected for small distances and that the p-wave contribution (within local optics) is proportional to $1/d^2$ for small distances. However, for the very high-resistivity materials, the p-wave contribution becomes much more important, and a crossover to a $1/d^2$ dependence of $(J_z)_\mathrm{p}$ is observed at very small separations d. This is illustrated in figures 4(b) and 4(c), which have been calculated with the same parameters as for figure 4(a), except that the electron mean free path has been reduced from $l = 560$ Å (the electron mean free

path for silver at room temperature) to 20 Å (roughly the electron mean free path in lead at room temperature) (figure 4(b)) and 3.4 Å (of order the lattice constant, representing the minimum possible mean free path) (figure 4(c)). Note that when l decreases, the p contribution to the heat transfer increases while the s contribution decreases. Since the mean free path cannot be much smaller than the lattice constant, the result in figure 4(c) represents the largest possible p-wave contribution for normal metals. However, the p-wave contribution may be even larger for other materials, e.g. semimetals, with lower carrier concentration than in normal metals. This fact has already been pointed out by Pendry [29]: the p-wave contribution for short distances is expected to be maximal when the function

$$\mathrm{Im}\, R_{\mathrm{p}} \approx \mathrm{Im}\,\frac{\epsilon-1}{\epsilon+1} = \mathrm{Im}\left[1-2\frac{\omega}{\omega_p}\left(\frac{\omega}{\omega_p}+\frac{\mathrm{i}}{\omega_p\tau}\right)\right]^{-1}$$

is maximal with respect to variations in $1/\tau$. This gives

$$\omega_p\tau = \frac{2k_BT}{\hbar\omega_p}$$

where we have used that typical frequencies $\omega \sim k_BT/\hbar$. Since the DC resistivity $\rho = 4\pi/(\omega_p^2\tau)$, we get (at room temperature) $\rho \approx 2\pi\hbar/k_BT \approx 0.14\ \Omega$ cm.

7. Summary and conclusions

We have used a general theory of a fluctuating electromagnetic field to calculate the frictional drag force between 2D and 3D electron systems. The separation d between the parallel electron layers is assumed to be so large that the only interaction between the layers is via the electromagnetic field associated with *thermal* and *quantum* fluctuations in the layers; the resulting friction force can be considered as the dissipative part of the van der Waals interaction. A general formula has been obtained, in which the frictional drag force is expressed through the electromagnetic reflection coefficients for s and p waves. We have found that the non-retarded Coulomb interaction, connected with evanescent p-polarized waves, is the dominant process for small layer separations and small electron densities. For high electron densities, retardation effects (associated with evanescent s waves) become very important, and we have suggested a new experiment, involving thin metallic films, in which the theory can be tested. We have shown that retardation effects are even more important for interaction between 3D electron systems. For very large separations the interaction is dominated by the travelling electromagnetic waves, which result from black-body radiation. However, the latter interaction appears negligible in comparison with phonon-mediated processes. Finally, we have pointed out the close relation between heat transfer and friction.

Acknowledgments

AIV acknowledges financial support from DFG. BNJP acknowledges financial support from BMBF.

References

[1] Pogrebinskii M B 1977 *Fiz. Tekh. Poluprov.* **11** 637 (Engl. Transl. 1977 *Sov. Phys.–Semicond.* **11** 372)
Price P J 1983 *Physica* B+C **117** 750
[2] Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1991 *Phys. Rev. Lett.* **66** 1216
Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1992 *Surf. Sci.* **263** 446
[3] Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1993 *Phys. Rev.* B **47** 12 957

Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1994 *Physica* B **197** 442
[4] Sivan U, Solomon P M and Shtrikman H 1992 *Phys. Rev. Lett.* **68** 1196
[5] Tso H C, Vasilopoulos P and Peeters F M 1993 *Phys. Rev. Lett.* **70** 2146
Świerkowski L and Gortel Z W 1995 *Phys. Rev. Lett.* **74** 3245
Vignale G and MacDonald A H 1996 *Phys. Rev. Lett.* **76** 2786
[6] Laikhman B and Solomon P M 1990 *Phys. Rev.* B **41** 9921
Boiko I I and Sirenko Yu M 1990 *Phys. Status Solidi* b **159** 805
Solomon P M and Laikhman B 1992 *Superlatt. Microstruct.* **10** 89
Maslov D I 1992 *Phys. Rev.* B **45** 1911
Cui H L, Lei X L and Horing N J 1993 *Superlatt. Microstruct.* **13** 221
Świerkowski L, Szymański J and Gortel Z W 1995 *Phys. Rev. Lett.* **74** 3245
[7] Mosko M, Cambel V and Moskova A 1992 *Phys. Rev.* B **46** 5012
Cambel V and Mosko M 1993 *Semicond. Sci. Technol.* **8** 364
[8] Tso H C, Vasilopoulos P and Peeters F M 1992 *Phys. Rev. Lett.* **68** 2516
[9] Zhang C and Takahashi Y 1993 *J. Phys.: Condens. Matter* **5** 5009
[10] Jauho A-P and Smith H 1993 *Phys. Rev.* B **47** 4420
[11] Kamenev A and Oreg Y 1995 *Phys. Rev.* B **52** 7516
[12] Flensberg K and Hu B Y-K 1994 *Phys. Rev. Lett.* **73** 3572
[13] Flensberg K, Hu B Y-K, Jauho A-P and Kinaret J 1995 *Phys. Rev.* B **52** 14 761
[14] Flensberg K and Hu B Y-K 1995 *Phys. Rev.* B **52** 14 796
[15] Świerkowski L, Szymański J and Gortel Z W 1997 *Phys. Rev.* B **55** 2280
[16] Bønsager M C, Flensberg K, Hu B Y-K and MacDonald A H 1998 *Phys. Rev.* B **57** 7085
[17] Lifshitz E M 1955 *Zh. Eksp. Teor. Fiz.* **29** 94 (Engl. Transl. 1956 *Sov. Phys.–JETP* **2** 73)
[18] Volokitin A I and Persson B N J 1999 *J. Phys.: Condens. Matter* **11** 345
Volokitin A I and Persson B N J 1998 *Phys. Low-Dimens. Struct.* **7+8** 17
[19] Persson B N J and Volokitin A I 2000 *Phys. Rev. Lett.* **84** 3504
[20] Rytov S M 1953 *Theory of Electrical Fluctuation and Thermal Radiation* (Moscow: Academy of Science of USSR Publishing)
[21] Dorofeev I, Fuchs H, Wenning G and Gotsman B 1999 *Phys. Rev. Lett.* **83** 2402
[22] Landau L D and Lifshitz E M 1970 *Statistical Physics* (Oxford: Pergamon)
[23] Landau L D and Lifshitz E M 1975 *Electrodynamics of Continuous Media* (Oxford: Pergamon)
[24] Stern F 1967 *Phys. Rev. Lett.* **18** 546
[25] Persson B N J 1984 *Solid State Commun.* **52** 811
[26] Persson B N J and Zhang Z 1998 *Phys. Rev.* B **57** 7327
[27] Ando T, Fowler A B and Stern F 1982 *Rev. Mod. Phys.* **54** 437
[28] Polder D and Van Hove M 1971 *Phys. Rev.* B **4** 3303
[29] Pendry J B 1999 *J. Phys.: Condens. Matter* **11** 6621
[30] Volokitin A I and Persson B N J 2001 to be published